MATH 115, 
PROBABILITY & STATISTICS 
N. PSOMAS 

Ch. 6.1 – HOMEWORK SOLUTIONS

6.13 Confidence interval mistakes and misunderstandings. Suppose 400 randomly selected 
alumni of the University of Okoboji were asked to rate the university’s counseling services on a 
1 to 10 scale. The sample mean (\( \bar{x} \)) was found to be 8.6. Assume that the population standard 
deviation is known to be \( \sigma = 2.0 \).

(a) Ima Bitlost computes the 95% confidence interval for the average satisfaction score as 
8.6 ± 1.96(2.0). What is her mistake?

(b) After correcting her mistake in part (a), she states “I am 95% confident that the sample 
mean falls between 8.404 and 8.796.” What is wrong with this statement?

(c) She quickly realizes her mistake in part (b) and instead states “The probability the true 
mean is between 8.404 and 8.796 is 0.95.” What misinterpretation is she making now?

(d) Finally, in her defense for using the Normal distribution to determine the confidence 
coefficient she says “Because the sample size is quite large, the population of alumni 
ratings will be approximately Normal.” Explain to Ima her misunderstanding and correct 
this statement.

Solution

6.13. (a) She did not divide the standard deviation by \sqrt{n}, \( n = 20 \). (b) Confidence intervals 
concern the population mean, not the sample mean. (The value of the sample mean is known to 
be 8.6; it is the population mean that we do not know.) (c) 95% is a confidence level, not a 
probability. Furthermore, it does not make sense to make probability statements about the 
population mean \( \mu \), which is an unknown constant (rather than a random quantity). (d) The large 
sample size does not affect the distribution of individual alumni ratings (the population 
distribution). The use of a Normal distribution is justified because the distribution of the sample 
mean is approximately Normal when the sample is large.

6.14 More confidence interval mistakes and misunderstandings. Suppose 100 randomly 
selected members of MySpace Karaoke were asked how much time they typically spend on the 
site during the week. The sample mean (\( \bar{x} \)) was found to be 4.2 hours. Assume that the 
population standard deviation is known to be \( \sigma = 2.5 \).

(a) Cary Oakey computes the 95% confidence interval for the average time on the site as 4.2 
\pm 1.96(2.5/100). What is his mistake?

(b) He corrects this mistake and then states “95% of the members spend between 3.71 and 
4.69 hours a week on the site.” What is wrong with his interpretation of this interval?

(c) The margin of error is slightly less than a half hour. To reduce this down to 15 minutes, 
Gary says the sample size needs to be doubled to 200. What is wrong with this statement?

Solution

6.14. (a) The standard deviation should be divided by \sqrt{100} = 10, not by 100. (b) The correct 
interpretation is that (with 95% confidence) the average time spent at the site is between 3.71 
and 4.69 hours.
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That is, the confidence interval is a statement about the population mean, not about the individual members. (c) To halve the margin of error, the sample size needs to be quadrupled, to about 400. (In fact, \( n = 385 \) would be enough.)

6.15 Importance of recreational sports. The National Intramural-Recreational Sports Association (NIRSA) performed a study to look at the value of recreational sports on college campuses.² A total of 2673 students were asked to indicate how important (on a 10-point scale) each of 21 factors was in terms of their college satisfaction and success. The factor “recreational sports and activities” resulted in a mean score of 7.5. Assume a standard deviation of 3.9.

(a) Give the margin of error and find the 95% confidence interval for this sample.

(b) Repeat these calculations for a 99% confidence interval. How do the results compare with those in part (a)?

Solution

6.15. (a) To estimate the mean importance of recreation to college satisfaction, the 95% confidence interval for \( \mu \) is \( 7.5 \pm 1.96 * (3.9 / \sqrt{2673}) = 7.5 \pm 0.1478 = 7.3522 \) to 7.6478

(b) The 99% confidence interval for \( \mu \) is \( 7.5 \pm 2.576 * (3.9 / \sqrt{2673}) = 7.5 \pm 0.1943 \) = 7.3057 to 7.6943

6.19 Populations sampled and margins of error. Consider the following two scenarios. (A) Take a simple random sample of 100 sophomore students at your college or university. (B) Take a simple random sample of 100 sophomore students in your major at your college or university. For each of these samples you will record the amount spent on textbooks used for classes during the fall semester. Which sample should have the smaller margin of error? Explain your answer.

Solution

6.19. Scenario B has a smaller margin of error. Both samples would have the same value of \( Z = (1.96) \), but the value of \( \sigma \) would be smaller for (B) because we would have less variability in textbook cost for students in a single major.

Note: Of course, at some schools, taking a sample of 100 sophomores in a given major is not possible. However, even if we sampled students from a number of institutions, we still might expect less variability within a given major than from a broader cross-section.

6.21 Consumption of sugar-sweetened beverages. A recent study estimated the U.S. per capita consumption of sugar-sweetened beverages among adults 20 to 44 years of age to be 289 kcal/day with a standard deviation of the mean equal to 7 kcal/day.¹⁰

(a) The 68–95–99.7 rule says that the probability is about 0.95 that \( \bar{x} \) is within _______ kcals/day of the population mean \( \mu \). Fill in the blank.

(b) About 95% of all samples will capture the true mean of kcals consumed per day in the interval \( \bar{x} \) plus or minus _______ kcals/day. Fill in the blank.
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Solution

6.21. (a) “The probability is about 0.95 that \( \bar{x} \) is within 14 kcal/day of \( \mu \)” (because 14 is two standard deviations). (b) This is simply another way of understanding the statement from part (a): If \( |\bar{x} - \mu| \) is less than 14 kcal/day 95% of the time, then “about 95% of all samples will capture the true mean \( \mu \) in the interval \( \bar{x} \) plus or minus 14 kcal/day.”

6.22 Apartment rental rates. You want to rent an unfurnished one-bedroom apartment in Dallas next year. The mean monthly rent for a random sample of 10 apartments advertised in the local newspaper is $980. Assume the monthly rents in Dallas follow a Normal distribution with a standard deviation of $290. Find a 95% confidence interval for the mean monthly rent for unfurnished one-bedroom apartments available for rent in this community.

Solution

6.22. For the mean monthly rent for unfurnished one-bedroom apartments in Dallas, the 95% confidence interval for \( \mu \) is $980 \pm 1.96 \times \frac{290}{\sqrt{10}} = $980 \pm $179.74 = $800.26 to $1159.74

6.25 Average hours per week on the Internet. The Student Monitor surveys 1200 undergraduates from 100 colleges semiannually to understand trends among college students. Recently, the Student Monitor reported that the average amount of time spent per week on the Internet was 19.0 hours. Assume that the standard deviation is 5.5 hours.

(a) Give a 95% confidence interval for the mean time spent per week on the Internet.

(b) Is it true that 95% of the students surveyed reported weekly times that lie in the interval you found in part (a)? Explain your answer.

Solution

6.25. (a) For the mean number of hours spent on the Internet, the 95% confidence interval for \( \mu \) is $19 \pm 1.96 \times 5.5/\sqrt{1200} = 19 \pm 0.3112 = 18.6888$ to $19.3112$ hours (b) No; this is a range of values for the mean time spent, not for individual times.
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6.28 Fuel efficiency. Computers in some vehicles calculate various quantities related to performance. One of these is the fuel efficiency, or gas mileage, usually expressed as miles per gallon (mpg). For one vehicle equipped in this way, the mpg were recorded each time the gas tank was filled, and the computer was then reset. Here are the mpg values for a random sample of 20 of these records:

<table>
<thead>
<tr>
<th>41.5</th>
<th>50.7</th>
<th>36.6</th>
<th>37.3</th>
<th>34.2</th>
<th>45.0</th>
<th>48.0</th>
<th>43.2</th>
<th>47.7</th>
<th>42.2</th>
</tr>
</thead>
<tbody>
<tr>
<td>43.2</td>
<td>44.6</td>
<td>48.4</td>
<td>46.4</td>
<td>46.8</td>
<td>39.2</td>
<td>37.3</td>
<td>43.5</td>
<td>44.3</td>
<td>43.3</td>
</tr>
</tbody>
</table>

Suppose that the standard deviation is known to be $\sigma = 3.5$ mpg.

(a) What is $\sigma_{\bar{x}}$, the standard deviation of $\bar{x}$?

(b) Examine the data for skewness and other signs of non-Normality. Show your plots and numerical summaries. Do you think it is reasonable to construct a confidence interval based on the Normal distribution? Explain your answer.

(c) Give a 95% confidence interval for $\mu$, the mean mpg for this vehicle.

Solution

6.28. (a) The standard deviation of the mean is $\sigma_{\bar{x}} = \frac{3.5}{\sqrt{20}} \approx 0.7826$ mpg. (b) A stemplot (right) does not suggest any severe deviations from Normality. The mean of the 20 numbers in the sample is $\bar{x} = 43.17$ mpg. (c) If $\mu$ is the population mean fuel efficiency, the 95% confidence interval for $\mu$ is

$$43.17 \pm 1.96 \left( \frac{3.5}{\sqrt{20}} \right) = 43.17 \pm 1.5339 = 41.6361 \text{ to } 44.7039 \text{ mpg}$$

6.34 Accuracy of a laboratory scale. To assess the accuracy of a laboratory scale, a standard weight known to weigh 10 grams is weighed repeatedly. The scale readings are Normally distributed with unknown mean (this mean is 10 grams if the scale has no bias). The standard deviation of the scale readings is known to be 0.0002 gram.

(a) The weight is measured five times. The mean result is 10.0023 grams. Give a 98% confidence interval for the mean of repeated measurements of the weight.

(b) How many measurements must be averaged to get a margin of error of $\pm 0.0001$ with 98% confidence?
Solution

6.34. (a) For the mean of all repeated measurements, the 98% confidence interval for $\mu$ is

$$10.0023 \pm 2.326 \left( \frac{0.0002}{\sqrt{5}} \right) = 10.0023 \pm 0.0002 = 10.0021 \text{ to } 10.0025 \text{ g}$$

(b) \( n = \left( \frac{(2.326)(0.0002)}{0.0001} \right)^2 \approx 21.64 \) — take \( n = 22 \).

6.35 More than one confidence interval. As we prepare to take a sample and compute a 95% confidence interval, we know that the probability that the interval we compute will cover the parameter is 0.95. That’s the meaning of 95% confidence. If we use several such intervals, however, our confidence that all of them give correct results is less than 95%. Suppose we take independent samples each month for five months and report a 95% confidence interval for each set of data.

(a) What is the probability that all five intervals cover the true means? This probability (expressed as a percent) is our overall confidence level for the five simultaneous statements.

(b) What is the probability that at least four of the five intervals cover the true means?

Solution

6.35. The number of hits has a binomial distribution with parameters \( n = 5 \) and \( p = 0.95 \), so the number of misses is binomial with \( n = 5 \) and \( p = 0.05 \). We can therefore use Table C or TI-83's `binompdf(n, p, k)` or `binomcdf(n, p, k)` functions to answer these questions. (a) The probability that all cover their means is 0.95$^5 = 0.7738$. (Or use Table C to find the probability of 0 misses.) (b) The probability that at least four intervals cover their means is 0.95$^5 + 5(0.05)(0.95^4)$ = 0.9774. (Or use Table C to find the probability of 0 or 1 misses.)

Using the TI-83:

(a) `binompdf(5, 0.95, 5) = 0.7738`

(b) `binompdf(5, 0.95, 5) + binompdf(5, 0.95, 4) = 0.7738 + 0.2036 = 0.9774`